**PROBABILITY DISTRIBUTION OF DISCRETE RANDOM VARIABLES**

**Glossary**

**Random Variable**: A variable, denoted by x, whose value is determined by the outcome of a random experiment. Also called a chance variable.

**Discrete random variable**: A random variable whose values are countable.

**Continuous random variable**: A random variable than can assume any value in one or more intervals.

**Probability distribution of a discrete random variable**: A list of all the possible values that a discrete random variable can assume and their corresponding probabilities.

**Mean of a discrete random variable**: The mean of a discrete random variable x is the value that is expected to occur per repetition, on average, if an experiment is performed a large number of times, the discrete random variable is also called its expected value.

**Standard deviation of a discrete random variable**: A measure of spread of the probability distribution of a discrete random variable.

**Parameter**: A summary measure calculated for population data.

**Binomial experiment**: An experiment that contains n identical trials such that each of these n trials has only two possible outcomes, the probabilities of these two outcomes remain constant for each trial and the trials are independent.

**Binomial parameters**: The total trials n and the probability of success p for the binomial probability distribution.

**Poisson probability distribution**: The probability distribution that gives the probability of x occurrences in an interval when the average occurrences in that interval are .

**Poisson parameters**: The average occurrences, denoted by , during an interval for a Poisson probability distribution.

**Probability Distributions of Discrete Random Variables**

1. What name is given to a table that lists all the values that a discrete random variable x can assume and their corresponding probabilities?
2. Classify each of the following random variables as discrete or continuous.
3. The time left on a parking meter
4. The total pounds of fish caught on a fishing trip
5. The number of gumballs in a vending machine
6. The time spent by a physician examining a patient
7. A household can watch news on any of the three networks – ABC, CBS, or NBC. On a certain day, five households randomly and independently decide which channel to watch. Let x be the number of households among these five that decide to watch news on ABC. Is x a discrete or a continuous random variables? Explain.
8. The following table gives the probability distribution of a discrete random variable *x*.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| *P(x)* | 0.11 | 0.19 | 0.28 | 0.15 | 0.12 | 0.09 | 0.06 |

Find the following probabilities.

1. *P(x=3)*
2. *P(x ≤ 2)*
3. *P(x ≥ 3)*
4. *P(1 ≤ x ≤ 4)*
5. Probability that x assumes a value less than 4
6. Probability that x assumes a value greater than 2
7. Probability that x assumes a value in the interval 2 to 5
8. Probability that x assumes a value between 2 to 5
9. Stanley Hook, a veteran baseball umpire, is not tolerant of players or managers who dispute his calls. Let *x* be the number of players and managers ejected from games by Hook during a week. The following table lists the probability distribution of *x*.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| *P(x)* | 0.10 | 0.18 | 0.23 | 0.25 | 0.14 | 0.07 | 0.03 |

1. Draw a bar graph for this probability distribution.
2. Determine the probability that the number of players ejected by Hook during a given week is
3. Exactly 3
4. At least 4
5. Less than 3
6. 2 to 5
7. A consumer agency surveyed all 2500 families living in a small town to collect data on the number of television sets owned by them. The following table lists the frequency distribution of the data collected by this agency.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Number of TV sets owned | 0 | 1 | 2 | 3 | 4 |
| Number of families | 120 | 970 | 730 | 410 | 270 |

1. Construct a probability distribution table for the numbers of television sets owned by these families. Draw a bar graph of the probability distribution.
2. Are the probabilities listed in the table of part a exact or approximate probabilities of various outcomes? Explain.
3. Let x denote the number of television sets owned by a randomly selected family from this town. Find the following probabilities.
4. P(x = 1)
5. P(x > 2)
6. P(x ≤ 1)
7. P(1 ≤ x ≤ 3)
8. According to a survey, 30% of adults are against using animals for research. Assume that this result holds true for the current population of all adults. Let x be the number of adults who are against using animals for research in a random sample of two adults. Obtain the probability distribution of x. Draw a tree diagram for this problem.
9. Find the mean and standard deviation for each of the following probability distributions.

|  |  |
| --- | --- |
| *x* | *P(x)* |
| 0 | 0.16 |
| 1 | 0.27 |
| 2 | 0.39 |
| 3 | 0.18 |

1. Let x be a discrete random variable that possesses a binomial distribution. Using the binomial formula, find the following probabilities.
2. *P(x = 5)* for *n* = 8 and *p* = 0.70
3. *P(x = 3)* for *n* = 4 and *p* = 0.40
4. *P(x =2)* for *n* = 6 and *p* = 0.30

Verify your answers by using Table I of Appendix C.

1. Let x be a discrete random variable that possesses a binomial distribution.
2. Using Table I of Appendix C, write the probability distribution of x for n = 7 and p = 0.30 and graph it.
3. What are the mean and standard deviation of the probability distribution developed in part a?
4. Magnetic resonance imaging (MRI) is a process that produces internal body images using a strong magnetic field. Some patients become claustrophobic and require sedation because they are required to lie within a small, enclosed space during the MRI test. Suppose that 20% of all patients undergoing MRI testing require sedation due to claustrophobia. IF five patients are selected at random, find the probability that the number of patients in these five who require sedation is
5. Exactly 2
6. None
7. Exactly 4
8. An office supply company conducted a survey before marketing a new paper shredder designed for home use. In the survey, 80% of the people who used the shredder were satisfied with it. Because of this high acceptance rate, the company decided to market the new shredder. Assume that 80% of all people who will use it will be satisfied. On a certain day, seven customers bought this shredder.
9. Let x denote the number of customers in this sample of seven who will be satisfied with this shredder. Using the binomial probabilities table (Table I, Appendix C), obtain the probability distribution of x and draw a graph of the probability distribution. Find the mean and standard deviation of x.
10. Using the probability distribution of part a, find the probability that exactly four of the seven customers will be satisfied.
11. A household receives an average of 1.7 pieces of junk mail per day. Find the probability that this household will receive exactly three pieces of junk mail n a certain day. Use the Poisson probability distribution formula.
12. On average, 20 households in 50 own answering machines.
13. Using the Poisson formula, find the probability that in a random sample of 50 households, exactly 25 will own answering machines.
14. Using the Poisson probabilities table, find the probability that the number of households in 50 who own answering machines is
15. At most 12
16. 13 to 17
17. At least 30
18. The student health center at a university treats an average of seven cases of mononucleosis per day during the week of final examinations.
19. Using the appropriate formula, find the probability that on a given day during the finals week exactly four cases of mononucleosis will be treated at this health center.
20. Using the appropriate probabilities table from Appendix C, find the probability that on a given day during the finals week the number of cases of mononucleosis treated at this health center will be
21. At least 7
22. At most 3
23. 2 to 5
24. An average of 4.8 customers to Columbia Savings and Loan every half hour.
    1. Find the probability that exactly two customers will come to this savings and loan during a given hour.
    2. Find the probability that during a given hour, the number of customers who will come to this savings and loan is
25. 2 or fewer

ii. 10 or more